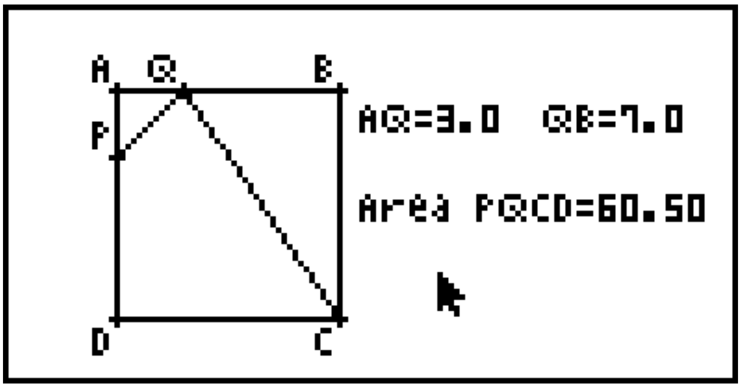
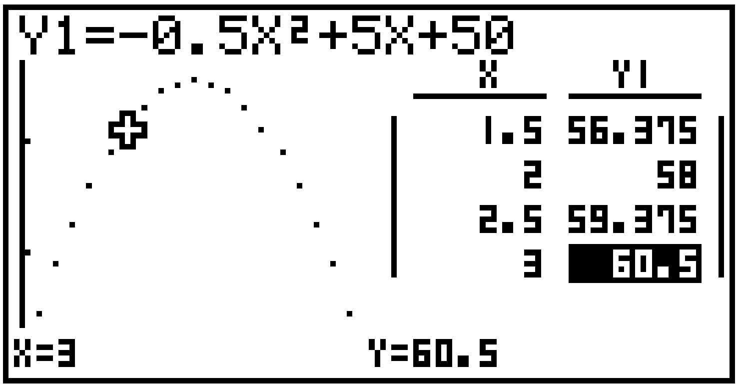


As BIG as can be?



Prince Alfred College Founded 1869

Noel Baker Centre for School Mathematics

CASIO®

As BIG as can be?

Version 1.00 – February 2008.

Written by Anthony Harradine and Alastair Lupton

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Index

Section Page

Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

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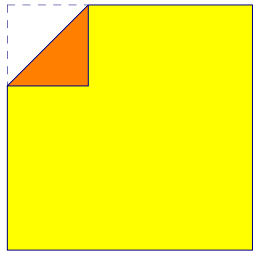
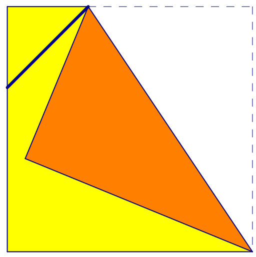
1. Stenduser: Is it really as BIG as it can be?

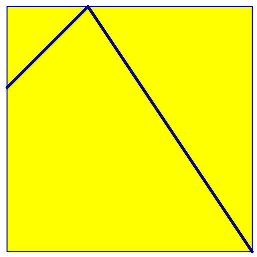
1.1 A little bit of paper folding



Take a 10 cm square piece of paper (yellow would be nice). Fold the top left hand corner so that its edges are parallel to the sides of the original square. You need **not** fold in the exact position shown.



Now fold this corner back, and make a second fold, from the top of the first fold to the bottom right hand corner of the square.



Folding back this second fold should result in a square that is divided by two fold lines into a quadrilateral and two right-angled triangles.

Compare your quadrilateral with the one above and with those of other students.

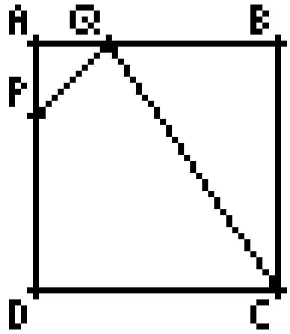
Discuss the following questions,

Are they all the same?

Do they all have the same area?

If not, which one appears to have the greatest area?

1.2 Investigating areas.



1. If the 10 cm square in question is labelled as shown right, explain how you would find the area of the quadrilateral PQCD.

2. Imagine that the paper folding activity was repeated 10 times with differing positions of point Q. Copy and complete the following table to investigate the affect that this variation has on the area of the quadrilateral PQCD.

Distance AQ	Distance BQ	Area ΔPAQ	Area ΔQBC	Area of PQCD
0 cm	10 cm	0 cm^2	50 cm^2	50 cm^2
1 cm				
2 cm				
3 cm				
4 cm				
5 cm				
6 cm				
7 cm				
8 cm				
9 cm				
10 cm				

3. Using the values in the above table as a starting point, draw an accurate graph of the area of PQCD against the distance AQ.

4. Describe what your graph illustrates about the variation in the area of PQCD as AQ varies.

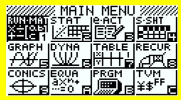
5. What do you *think* is the maximum area of PQCD and the corresponding length of AQ?

6. What do you *think* is the minimum area of PQCD and the corresponding length(s) of AQ?

1.3 Capturing the infinite …

Open the Geometry file entitled `OPTQUAD` on your CASIO *9860G AU*.

Run the animation that has been built into this geometric construction.



16.1

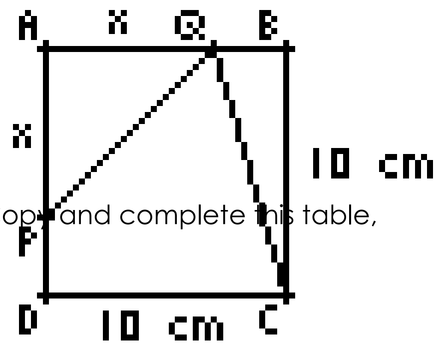
1. How many quadrilaterals PQCD are possible?

2. The variation in what quantity determines PQCD and its area?

If we define x as a variable representing all of the possible values of length AQ,

3. What range of values can x take?

4. Write down an expression, in terms of x, for the area of ΔAQP.



5. Copy and complete this table,

Distance AQ	Distance BQ
1 cm	10 – 1 = 9 cm
2 cm	10 – 2 = 8 cm
3 cm	
4 cm	
5 cm	
x cm	

6. Use this expression for BQ to write down an expression for the area of ΔQBC in terms of x.

7. Using your previous answers, write down an expression for the area of quadrilateral PQCD in terms of x.

8. Find the values of x that satisfy the equation $-\frac{1}{2}x^2 + 5x + 50 = 50$.

9. Explain the significance of the equation and the resultant solutions in part 8.

10. Prove that the value you conjectured to be the maximum area of PQCD (in section 1.2 part 5) is, in fact, as big as the quadrilateral can be![1]

11. Find the maximum area of PQCD if this paper folding activity were undertaken using a square of side length *a* units.[1]

12. Find the maximum area of PQCD if this paper folding activity were undertaken using a rectangular piece of paper.[1]

---

[1] If you cannot complete questions 10, 11 and 12 then … you need to know more!
You will get a chance to return to these tasks when you are better equipped

2. *Some terminology.*

To help you work and learn in this area of mathematics it is important that you understand and are able to use correctly the following terms:

algebraic model **algebraic expression** **equation** **function**

In the *Stenduser* you derived (or developed) an **algebraic model** for the area of *all* possible quadrilaterals PQCD (i.e. for the "general case" quadrilateral).

An **algebraic model** is simply a collection of pro-numerals and numbers that are linked with desired operators (i.e. addition, multiplication etc.) and the equivalence relationship (=) to form *an equation that describes the relationship between associated quantities*. In our case, we developed an **algebraic model** that described

- how the area of the quadrilateral can be calculated, and
- how the area varies as the position of Q varies (i.e. as the length AQ varies).

The **algebraic model** derived in the *Stenduser* was $A = -\frac{1}{2}x^2 + 5x + 50$.

This is a model for the area, A in cm², of every quadrilateral PQCD formed by our method of folding a 10cm × 10cm paper square.

Other names for algebraic structures like our model are *rule*, *formula*, *algebraic representation* and *equation*. The terms rule and **equation** are also often used in describing relationships in mathematics that are not trying to model some situation like the area of our quadrilateral.

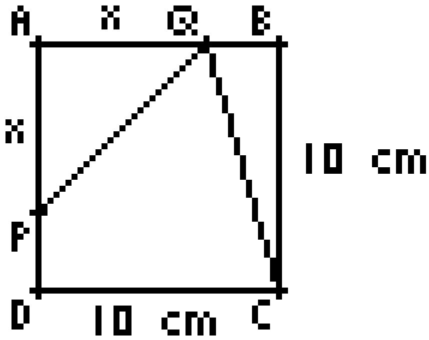
The right hand side of our **equation**, i.e. $-\frac{1}{2}x^2 + 5x + 50$ is called an **algebraic expression**. *Note that it contains no "equals sign"*.

Clearly the algebraic model $A = -\frac{1}{2}x^2 + 5x + 50$ will compute A values (output) for any given x value (input) or, to keep it in context, will compute the area of the quadrilateral PQCD for any given value of the length AQ.

In formal mathematics a rule like $A = -\frac{1}{2}x^2 + 5x + 50$ that involves a value (x) being operated on in some way to produce a unique value (A) is called a **function**.

The output value (A in this case) is normally reported with the input value (x in this case) as an ordered pair (x , A).

3. *Representing…*



The *Stenduser* has a **geometric representation**.

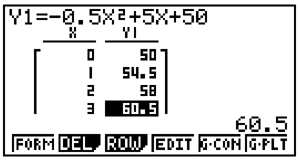
Based on this representation, our task is to find the maximum area of the quadrilateral PQCD if Q can take any position on AB.

The *Stenduser* also has an **algebraic representation**

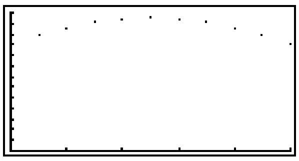
$$A = -\frac{1}{2}x^2 + 5x + 50$$

In this representation our task is to find the ordered pair (x , A) such that A takes its largest possible value.

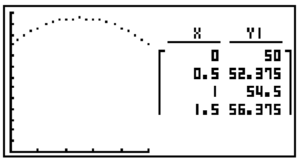
We can make a **tabular representation** of our *Stenduser* (or any other function) by documenting *some* of the ordered pairs that result from this function.



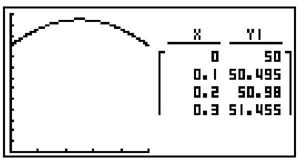
We can make a **graphical representation** of our *Stenduser* (or any other function) by plotting some of these ordered pairs on a Cartesian Plane where the horizontal axis represents the input variable (i.e. x) and the vertical axis represents the output variable (i.e. A)



Of course, there are more ordered pairs that can be generated by our function than those that are shown in the table or the graph above. Putting in some "in between" ordered pairs gives,

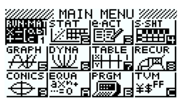


Adding some more ordered pairs gives the impression of a smooth curve.



A graphical representation of a function consisting of a *continuous line* acknowledges that the function consists of *an infinite set of ordered pairs*.

Care should be taken when deciding whether to represent a function as a selection of points or as a continuous line. Often only one will be appropriate.



16.2

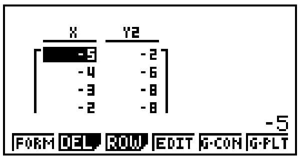
4. *The road to proof.*

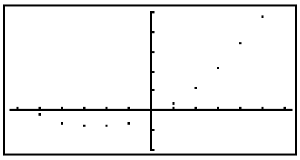
At this stage you probably think you know the maximum area of the quadrilateral PQCD in the *Stenduser*. You may be right but can you be sure beyond any doubt? *Can you prove it?*

1. Use your graphic calculator to make a tabular representation and a graphical representation of the function,

   $y = x^2 + 5x - 2$ for $-5 \le x \le 5$ and integer values of x.

   You should achieve the following





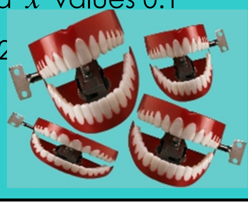
2. It appears that y has a minimum value in this function. What is it?

   What is the corresponding x value?

Can you be sure of your answers, beyond any doubt?

3. Describe the method you used to determine your answer to question 2.

Harry said he could not be sure beyond any doubt that the value that he thought was the minimum value was *actually* the minimum value. He claimed that there were too many points to check to make sure he was right. He thought the minimum y value was –8.25 when x was –2.5, but said that even if he checked x values 0.1 either side of –2.5, there were values 0.01 and 0.001 either side of –2 needed to be checked and so on…



Discuss Harry's thinking

4. For each of the following functions, determine, as best you can, whether y has a maximum or minimum value, the actual maximum or minimum value and the corresponding value of x.

   a. $y = x^2 + 4x - 2$ b. $y = -x^2 + 8x - 2$ c. $y = 3x^2 + 7x - 2$

   d. $y = (x+3)^2 + 5$ e. $y = -2(x-2)^2 - 7$ f. $y = -(x+8)^2 + 2$

Summarise your results in a table like the following:

Function	Maximum/Minimum	Max. or Min. value of y.	Corresponding value of x.
$y = x^2 + 4x - 2$			
…			

Can you be sure, beyond any doubt, of the entries in your table?

5. Look carefully at your table. What patterns or links do you observe? Make conjecture(s) based on your observations.

Being sure beyond any doubt requires you to prove a statement.

Let us start with the function found in Part d of Question 4, $y=(x+3)^2+5$.

I believe that y has a minimum value of 5 when x = –3.

To prove this I could argue as follows:

$x^2 \geq 0$ for all x (as x^2 is a square number which are always non-negative)

$\Rightarrow (x+3)^2 \geq 0$ for all x

$\Rightarrow (x+3)^2+5 \geq 5$ for all x (adding 5 to both sides of the inequality)

$\Rightarrow$ y has a minimum value of 5 (as $y=(x+3)^2+5$)

Now the minimum occurs when $(x+3)^2=0$, i.e. when x = –3.

$\therefore$ y has a minimum value of 5 when x = –3.

6. For each of the following functions, state whether y has a maximum or minimum value, the actual maximum or minimum value and the corresponding value of x. *Prove each of your statements.*

a. $y=(x+1)^2+11$ b. $y=(x-2)^2-15$ c. $y=(x+12)^2-9$

d. $y=-(x+1)^2+4$ e. $y=-(x-3)^2-9$ f. $y=x^2-6x+9$

g. $y=x^2-14x+50$ h. $y=x^2+6x-2$

5. Forms of Quadratic Functions

How did you cope with Parts f, g and h of Question 6?

You probably had more difficulty with these questions were in a different *form* to the other parts of Question 6.

All the functions that we have been dealing with in this unit have been *quadratic functions*, in one form or another. A quadratic function is one in which the highest exponent value of the unknown to be operated on (usually x) is 2. All other exponent values must be non-negative integers, so they must be either 1 or 0.

Therefore, all quadratic functions have the form:

$y = ax^2 + bx + c$ where a, b and c are constant values with $a \neq 0$.

This form is called *general form* of the quadratic function.

So, is $y = (x+1)^2 + 11$ a quadratic function?

If we expand and simplify its right-hand side we have:

$$\begin{aligned} & y = (x+1)^2 + 11 \\ \Rightarrow\ & y = x^2 + 2x + 1 + 11 \\ \Rightarrow\ & y = x^2 + 2x + 12 \end{aligned}$$

Hence we can see it is a quadratic, but was presented in a different form. You should be able to see that anything in the form

$$y = a(x+h)^2 + k$$

will expand to general form.

1. Expand and simplify $y = a(x+h)^2 + k$ to show it can be expressed in general form.

2. Use expansion and simplification to express each of the following quadratic functions in general form.

a.	$y = (x+3)^2 + 2$	b.	$y = (x-2)^2 + 1$	c.	$y = (x-5)^2 - 20$
d.	$y = -(x+1)^2 - 4$	e.	$y = 2(x-6)^2 - 50$	f.	$y = -3(x+4)^2 + 12$

6. *Turning Point form.*

Thinking back to our proofs of maximum or minimum values in Section 4, the arguments were based on quadratics in the form $y = a(x+h)^2 + k$.

This is often called the *turning point* or *vertex* form of the quadratic function.

The proof that $y = x^2 + 6x - 2$ has a minimum value of –11 when x = –3 is made possible by first changing this function from *general form* to *turning point form*.

To do this we have to <u>create</u> a square number, i.e. a term of the form $(x+h)^2$.

<u>This process is called *completing the square*.</u>

The square we will 'create' is $(x+3)^2$, as this has the expansion $(x+3)^2 = x^2 + 6x + 9$ (and thus contains the first two of the three terms in our quadratic).

$$\begin{aligned} & y = x^2 + 6x - 2 \\ \Rightarrow\ & y = x^2 + 6x + 9 - 2 - 9 \\ \Rightarrow\ & y = (x+3)^2 - 2 - 9 \\ \Rightarrow\ & y = (x+3)^2 - 11 \end{aligned}$$

For this reason, all we need to do to create this square is to add the third term ($+9$), but we must also take it away (from the -2) in order to maintain function equivalence.

Now that we have the quadratic in turning point form we can complete the following argument.

$x^2 \geq 0$ for all x (as x^2 is a square number which are always non-negative)
$\Rightarrow (x+3)^2 \geq 0$ for all x
$\Rightarrow (x+3)^2 - 11 \geq -11$ for all x (subtracting 11 from both sides of the inequality)
$\Rightarrow y$ has a minimum value of – 11.

Now the minimum occurs when $(x+3)^2 = 0$, i.e. when x = –3.
$\therefore\ y$ has a minimum value of – 11 when x = –3.

1. Change these quadratic functions from general form to turning point form and state the maximum or minimum value and the corresponding value of x.

a. $y = x^2 + 8x - 1$ b. $y = x^2 + 10x + 2$ c. $y = x^2 - 2x + 2$

d. $y = x^2 - 4x - 6$ e. $y = x^2 + 3x$ f. $y = x^2 - 5x + 40$

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7. *Further Completing the Square.*

Consider $x^2+6x+9=(x+3)^2$ and $2x^2+12x+18=2(x^2+6x+9)=2(x+3)^2$.

This suggests that if the co-efficient of x^2 is not 1 then it should be factored out before completing the square. For example,

$y=3x^2+5x-7$

$\Rightarrow y=3(x^2+\frac{5}{3}x)-7$ (Note: do *not* factor the constant term!)

$\Rightarrow y=3(x^2+\frac{5}{3}x+\left(\frac{5}{6}\right)^2-\left(\frac{5}{6}\right)^2)-7$ (As $\frac{5}{6}=\frac{5}{3}\div 2$)

$\Rightarrow y=3([x+\frac{5}{6}]^2-\left(\frac{5}{6}\right)^2)-7$

$\Rightarrow y=3(x+\frac{5}{6})^2-3\times\left(\frac{5}{6}\right)^2-7$

$\Rightarrow y=3(x+\frac{5}{6})^2-\frac{109}{12}$ (Your calculator may be helpful for this line)

The same method applies for negative and non-integer co-efficients of x^2, e.g.

$y=-\frac{1}{3}x^2+2x-2$

$\Rightarrow y=-\frac{1}{3}(x^2-6x)-2$ (Note: do *not* factor the constant term!)

$\Rightarrow y=-\frac{1}{3}(x^2-6x+9-9)-2$ (As $(-6\div 2)^2=9$)

$\Rightarrow y=-\frac{1}{3}([x-3]^2-9)-2$

$\Rightarrow y=-\frac{1}{3}(x-3)^2-\frac{1}{3}\times-9-2$

$\Rightarrow y=-\frac{1}{3}(x-3)^2+1$

1. Change these quadratic functions from general form into turning point form.

a.	$y=2x^2+8x-4$	b.	$y=5x^2+10x+3$	c.	$y=-2x^2+6x+2$
d.	$y=-3x^2-12x+1$	e.	$y=-2x^2+5x-7$	f.	$y=\frac{1}{2}x^2+3x+5$
g.	$y=\frac{1}{3}x^2-12x+6$	h.	$y=-\frac{1}{2}x^2+2x+2$	i.	$y=ax^2+bx+c$

Hint: persevere with part i, it will save a lot of time later!

2. If you have not yet done so, re-visit the *Stenduser* on page 7.

8. *The graphs of quadratic functions.*

8.1 Symmetry

1.

a. Describe what happens to the values of $y = x^2$ as x goes from 0 to $+\infty$ or "positive infinity" (i.e. increasingly larger values of x).

b. Describe what happens to the values of $y = x^2$ as x goes from 0 to $-\infty$ or "negative infinity" (i.e. increasingly smaller values of x).

c. How would these observations show themselves in the graph of $y = x^2$

2.

a. Describe what happens to the values of $y = (x-3)^2$ as x goes from 0 to $+\infty$ or "positive infinity" (i.e. increasingly larger values of x).

b. Describe what happens to the values of $y = (x-3)^2$ as x goes from 0 to $-\infty$ or "negative infinity" (i.e. increasingly smaller values of x).

c. How would these observations show themselves in the graph of $y = (x-3)^2$

3. Copy and complete the table of values for the two quadratics functions for $-2 \le x \le 5$ and hence plot the graphs "by hand".

x	-2	-1	0	1	2	3	4	5
$y = x^2$								
$y = (x-3)^2$								

4. What might mathematicians mean when they describe quadratic functions as "symmetric"?

8.2 Families of Quadratic Functions

1. Use your CASIO *9860G AU* to draw all of these graphs on the same axes:

$y = x^2$ $\quad y = 2x^2$ $\quad y = 5x^2$ $\quad y = \frac{3}{4}x^2$ $\quad y = -3x^2$

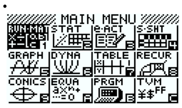
The set of *all* the possible functions of the above form is called a *family of quadratic functions*. This family can be described symbolically as $y = mx^2$, where *m* is an "unknown constant" that takes different values (i.e. 1, –3, 5, 7.2, 0 …).
By taking different values *m* "generates" the members of the family of quadratics.
Such an unknown constant is sometimes referred to as a *parameter*.

Use the examples of this family that you have graphed to make a conjecture about:

i. the properties the members of this family have in common;

ii. the way that members of this family differ from one another.

iii. whether or not there is an "odd function out" in this family



16.3

2

a. Investigate these families of functions in the same fashion

i. $y=(x-h)^2$ ii. $y=x^2+k$ iii. $y=m(x+3)^2+2$

In each case choose your own specific functions to plot, but be sure to include zero as well as some positive values and some negative values of the parameter.

b. Summarise what you have observed about the relationship between parameters and the corresponding families of quadratic functions.

3. Using your understanding developed in Question 2, graph these sets of quadratic functions on the same set of axes,

a. $y=x^2$, $y=(x+4)^2$, $y=(x+4)^2-2$, $y=\frac{1}{2}(x+4)^2-2$

b. $y=x^2$, $y=x^2+1$, $y=-x^2+1$, $y=-3x^2+1$

c. $y=x^2$, $y=(x-2)^2$, $y=(x-2)^2+5$, $y=-2(x-2)^2+5$

Check your graphs using your 9860.

4. Graphing some members of the family of quadratic functions

$$y=mx^2-2mx-3m$$

shows a particularly obvious property. Can you explain the reason for that property by looking at the general equation $y=mx^2-2mx-3m$?

8.3 Find a function…

1. Find the equation of a quadratic function whose graph never cuts the x–axis.

2. Find the equation of a quadratic function whose graph cuts the x–axis just once.

3. Find the equation of a quadratic function whose graph cuts the x–axis exactly twice.

4. Find the equation of a quadratic function whose graph cuts the x–axis three times.

5.

a. Find the equation of a quadratic function which passes through the point (5, 13).

b. Find the equation of a quadratic function which passes through the point (5, 13), and cuts the x-axis twice.

c. Find the equation of a quadratic function which passes through the point (5, 13), and cuts the x-axis once.

d. Find the equation of a quadratic function which passes through the point (5, 13), and has a maximum value of 14.

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9. *So … how am I going…?*

These questions cover many of the ideas and techniques that have been covered in this unit up to this point. Have a go at them to see how much you have learned.

1. Convert these quadratic functions to general form $y = ax^2 + bx + c$.

   a. $y = (x+7)^2 - 24$ b. $y = -2(x-1)^2 + 6$

2. Convert these quadratic function to turning point form $y = a(x+h)^2 + k$.

   a. $y = x^2 + 10x + 30$ b. $y = 4x^2 + 12x + 5$

3. For the quadratic function

   $y = -(x-3)^2 + 1$

   a. Write down the co-ordinates of this function's turning point

   b. Hence sketch the graph of this function on a co-ordinate axes.

4. For the quadratic functions given in this table, determine.

Quadratic Function	Do they have a maximum or a minimum value?	What is the maximum / minimum value?	When does this max. / min. value occur?	How many zeros do they have (do not find them)
$y = (x+2)^2 + 11$				
$y = -5(x-3.5)^2 + 2$				

5. a. Imagine that the graph of $y = mx^2$ is drawn for a specific positive value of m. If the value of m is doubled, describe the effect this has on the graph

   b. Imagine that the graph of $y = x^2 + k$ is drawn for a specific value of k. If the value of k is increased by 5 units, describe the effect this has on the graph

6. a. Convert the quadratic function $y = -\frac{1}{3}x^2 + 2x + 10$ into *turning point form*.

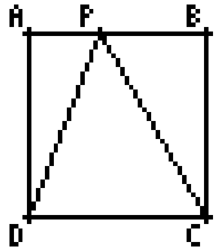
   b. It has been shown that the quadratic function,

   $A = -\frac{1}{2}x^2 + 5x + 50$ can be written as $A = -\frac{1}{2}(x-5)^2 + 62.5$.

   Provide a deductive argument proving that this function has a maximum value of $A = 62.5$ that occurs when $x = 5$

7. A point P is marked on the edge AB of a *10 cm by 10 cm* square ABCD.

a. If P is positioned so that AP = 4 cm (as shown)



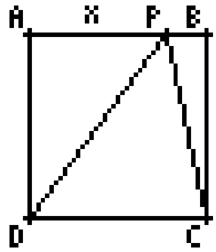
i. Write down the length of PB.

ii. Find the area of triangle APD

iii. Find the area of triangle PBC

iv. Find the sum of the areas of Δ APD + Δ PBC

b. Consider point P to be positioned so that AP = x cm



i. Write down an expression for the length of PB.

ii. Write down an expression for the area of Δ APD

iii. Write down an expression for the area of Δ PBC

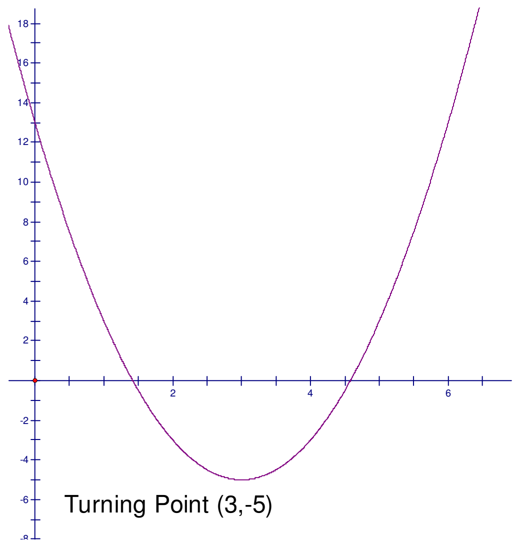
iv. Write down an expression for the sum of the areas of Δ APD + Δ PBC

c. What does your answer to part iv above tell you about Δ APD + Δ PBC? Provide reasoning to support your answer

d. Is there another way to arrive at your conclusion in part c?

10. *The Turning Point.*

The curved graph of a quadratic function is called a *parabola*.



In terms of the graph of a quadratic function, the maximum or minimum value corresponds to the *turning point* or *vertex* of the parabola.

Hence using the method of "completing the square" to write the general form $y = 2x^2 - 12x + 13$ of a quadratic function into the vertex form $y = 2(x-3)^2 - 5$ allows us to see that the vertex of its graph is at (3, –5).

An easier method than completing the square each time is to do it just once, on the general quadratic function

$$y = ax^2 + bx + c$$

and then to use it as a formula.

If you successfully completed Question 1 part i on page 14 you will know that

$$y = ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$$

and hence we can see that the vertex occurs at $\left(-\frac{b}{2a}, -\frac{b^2-4ac}{4a}\right)$

In our example $y = 2x^2 - 12x + 13$ we have $a = 2$, $b = -12$ and $c = 13$.

Hence the vertex occurs at

$$\left(-\frac{-12}{2\times 2}, -\frac{(-12)^2 - 4\times 2\times 13}{4\times 2}\right) = (3,-5)$$

1. Use this approach to find the turning points of each of the following functions:

a.	$y = x^2 + 3x - 4$	b.	$y = 5x^2 + 4x - 2$	c.	$y = -x^2 + 3x + 2$
d.	$y = \frac{1}{2}x^2 + 2x + 3$	e.	$y = -2x^2 + \frac{2}{3}x - \frac{2}{7}$	f.	$y = 2x^2 + 4x + k$

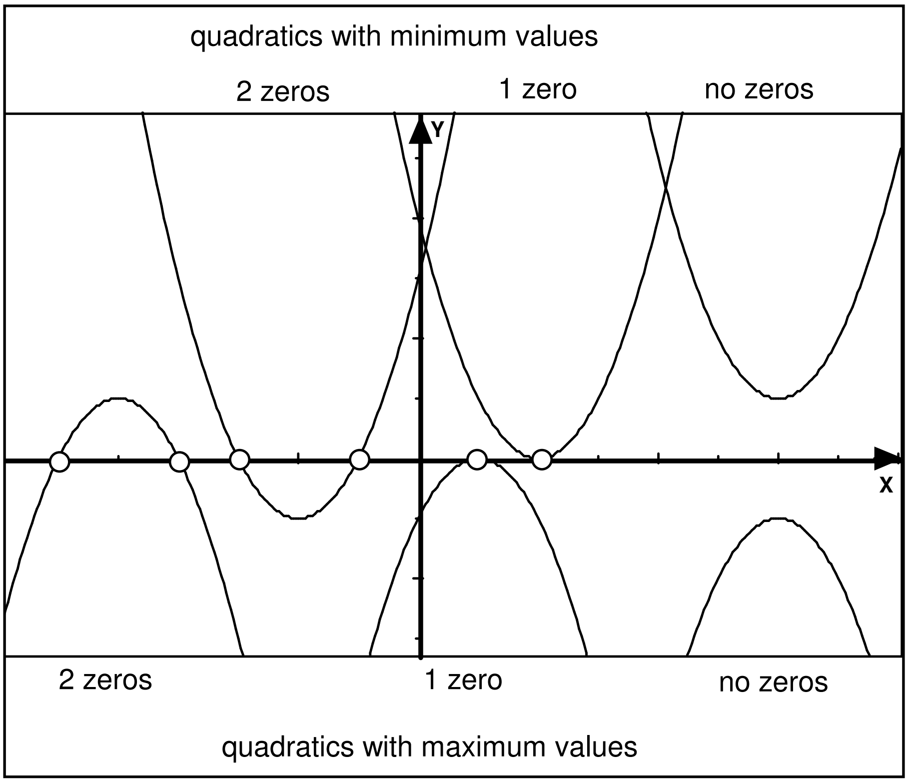
11. Zeros

A *zero* of a function is the input value (often denoted as x) which makes the output of the function (usually y) equal to zero.

A quadratic function can have:

- two zeros – e.g. $y = x^2 - 4x + 3$ has zeros, $x = 1$ and $x = 3$.
- one zero – e.g. $y = x^2 - 4x + 4$ has one zero, $x = 2$.
- no zeros – e.g. $y = x^2 - 4x + 4$ has no zeros .

On the graph of a quadratic function the zeros correspond to the points where $y = 0$, i.e. the points where the graph crosses the x-axis. This is called the x-axis intercept or the *root* of the function. There are *six* possible cases for quadratic functions:



The graphs above illustrate that:

- a quadratic function with a *negative minimum value*, e.g. $y = (x+3)^2 - 5$, or a *positive maximum value*, e.g. $y = -(x+3)^2 + 5$, will have *two zeros*.
- a quadratic function with a *minimum value of zero*, e.g. $y = (x+3)^2$, or a *maximum value of zero*, e.g. $y = -(x+3)^2$ will have just that *one zero*.
- a quadratic function with a *positive minimum value*, e.g. $y = (x+3)^2 + 5$, or a *negative maximum value*, e.g. $y = -(x+3)^2 - 5$ will have *no zeros*.

12. But what sort of zeros?

1. For the following quadratic functions, write down their zero(s) – use a table to record your results.

$y = x^2 - 4x + 6$	$y = x^2 - 4x$
$y = x^2 - 4x + 5$	$y = x^2 - 4x - 1$
$y = x^2 - 4x + 4$	$y = x^2 - 4x - 2$
$y = x^2 - 4x + 3$	$y = x^2 - 4x - 3$
$y = x^2 - 4x + 2$	$y = x^2 - 4x - 4$
$y = x^2 - 4x + 1$	$y = x^2 - 4x - 5$

These functions are specific members of the family of quadratic functions that could be described as all the functions $y = x^2 - 4x + c$ where c is some integer value.

2.
   a. Describe what you notice about the zeros of these members of this family of quadratics.?
   b. How many members of this family have integer zeros?
      Can you explain why this is so?
   c. What is the 'next' member of the family that has integer zeros?
   d. Can you write down the next four members of the family that have integer zeros?
      What will their zeros be?
   e. Why is it that some members of the family have integer zeros and the other members of the family do not?

Finding integer zeros.

3. Consider the algebraic identity $x^2 - 7x - 18 = (x + 2)(x - 9)$.
   a. Clearly show that $x = -2$ is a zero of *both sides* of this identity.
   b. Suggest a second zero for the two sides of this identity.
   c. Check that your suggestion is correct.

4. Write down a similar identity for the right hand side of $y = x^2 + 7x + 12$ and hence write down its zeros.
   a. Repeat for the quadratic function $y = x^2 - 3x - 10$.
   b. Repeat for the quadratic function $y = x^2 - 2x + 1$.
   c. Repeat for the quadratic function $y = x^2 + 5x - 6$.
   d. Repeat for the quadratic function $y = x^2 + x - 30$.
   e. Repeat for the quadratic function $y = 2x^2 + 9x + 9$.
   f. Repeat for the quadratic function $y = 3x^2 + 11x - 14$.
   g. Repeat for the quadratic function $y = -x^2 - 11x - 10$.
   h. Repeat for the quadratic function $y = -4x^2 - 19x + 5$.

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Two zeros – many quadratics

5. Consider the values $x = -1$ and $x = 2$ as the zeros of a quadratic function.

   a. Write down a possible such quadratic function.

   b. Write down another possible quadratic function.

   c. How many possible quadratics are there?

   d. Describe the family of quadratics that have $x = -1$ and $x = 2$ as their zeros.

6.

   a. Write down a quadratic with zeros $x = 4$ and $x = 6$

   b. Write down *all* quadratic with zeros $x = 4$ and $x = 6$ (one line answer)

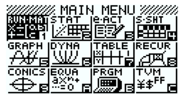
   c. Write down a quadratic with zeros $x = 4$ and $x = 6$ and a y-intercept of 8.

   d. Write down a quadratic with zeros $x = 4$ and $x = 6$ and a maximum value of $y = 1$.

   e. Write down a quadratic with zeros $x = 4$ and $x = 6$ that passes through the point $(2, -24)$.

   f. Draw the graph of the functions that you wrote down in parts c, part d and part e on the same axes.

13. *Solving quadratics.*



16.4

From what you have already seen there is much to know about quadratic functions and many ways to obtain this information.

The vertex of a quadratic function.

Also called the *turning point* of the function, this is the point where the function reaches its greatest or least value (its *maxima* or *minima*).

The vertex $(-h, k)$ can be found if a quadratic is written in the form $y = a(x+h)^2 + k$.

This form can be obtained from general form by *completing the square*.

It can also be obtained by using the `G-Solv` menu of your CASIO *fx-9860G AU*, assuming that you have drawn a graph in GRAPH that contains the vertex.

Toni thinks she has a method to find the location of the vertex of a quadratic based on the location of its zeros.

Can you work out what her method would be?

What property of the quadratic function does this method rely on?

1. Find the turning point of the following quadratic functions

a.	$y = x^2 - 10x + 21$	b.	$y = x^2 - 6x + 9$	c.	$y = 2(x-1)^2 + 3$
d.	$y = (x-2)(x+5)$	e.	$y = 2x^2 + 8x - 24$	f.	$y = (x+\frac{1}{3})^2$
g.	$y = x^2 - x - 20$	h.	$y = -x^2 + 4x - 8$	i.	$y = -4x^2 + 8x + 5$

The zeros of a quadratic function.

Finding the zeros of a quadratic function means determining the value(s) of x for which the quadratic function takes a zero (y) value (i.e. the x for which $y = 0$).[2]

Not surprisingly this can be represented, solving the equation equals to zero.

For	$y = (x-2)^2 - 5$	$y = x^2 + 6x - 16$
Solve	$(x-2)^2 - 5 = 0$	$x^2 + 6x - 16 = 0$
	$\therefore (x-2)^2 = 5$	$\therefore (x+2)(x-8) = 0$
	$\therefore x - 2 = \pm\sqrt{5}$	$\therefore x = -2 \text{ or } x = 8$
	$\therefore x = 2 \pm \sqrt{5}$	*(by the Null Factor Law)*

[2] Note, finding these zeros equates to finding the x-intercepts of the graph of the quadratic function. This is sometimes referred to as finding the roots of the function.

This result can also be obtained using the **G-Solv** menu of your CASIO *fx-9860G AU*, assuming that you have drawn a graph in GRAPH that contains the zeros of the function. Note that, in the first example the zeros are found to be – 0.236 and 4.236, which are 3 decimal place approximations for the exact surd zeros $2-\sqrt{5}$ and $2+\sqrt{5}$. Sometimes the decimals are "good enough" but at other times we need to find the exact zeros, and so an algebraic method may be needed.

2. Find zeros of these quadratics *in two different ways* (including exact values)

a.	$y=x^2-10x+21$	b.	$y=x^2-6x+9$	c.	$y=(x-2)^2-5$
d.	$y=(x-2)(x+5)$	e.	$y=2x^2+8x-24$	f.	$y=(x+\frac{1}{3})^2$
g.	$y=x^2-x-20$	h.	$y=-x^2+4x-8$	i.	$y=-4x^2+8x+5$

If we call $y=a(x+p)(x+q)$ the *factored* form of a quadratic function then we can see that quadratic functions can exist in three different guises

- General Form $y=ax^2+bx+c$
- Turning Point Form $y=a(x+h)^2+k$
- Factored Form $y=a(x+p)(x+q)$

3. In which of these forms is it easiest to identify
   a. The vertex of the graph of the quadratic function?
   b. The zeros of the graph of the quadratic function?
   c. The y-intercept of the graph of the quadratic function?

The mathematics involved in finding the zeros of quadratic functions is also used in the solution of equations that involve quadratic expressions.

To solve	$x^2=3x-24$	$2(x+1)^2=2x+7$
Re-express	$\therefore x^2-3x+24=0$	$2x^2+4x+2=2x+7$ $2x^2+2x-5=0$
And find zeros	$\therefore (x-8)(x+3)=0$ $\therefore x=8 \text{ or } x=-3$	$2(x^2+x)-5=0$ $2[(x+\frac{1}{2})^2-\frac{1}{4}]-5=0$ $2(x+\frac{1}{2})^2-\frac{1}{2}-5=0$ $2(x+\frac{1}{2})^2=\frac{11}{2}$ $(x+\frac{1}{2})^2=\frac{11}{4}$ $x+\frac{1}{2}=\pm\sqrt{\frac{11}{4}}$ $x=-\frac{1}{2}\pm\frac{\sqrt{11}}{2}$

In both examples the quadratic in general form was obtained, factorisation was attempted then completing the square was undertaken where needed.

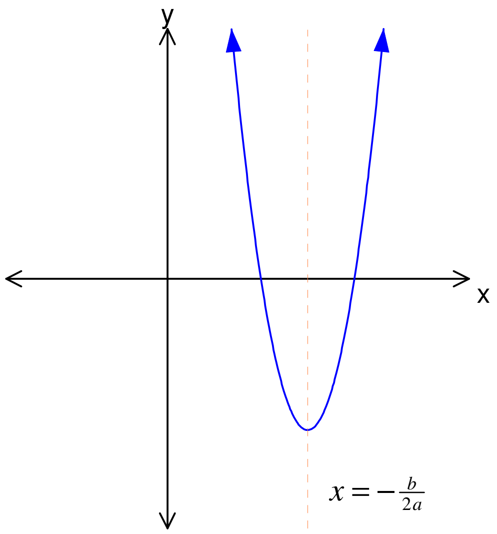
Clearly an approximate solution for the second example, obtained by graphing the quadratic in general form (the left hand side of line 3) and finding its zeros would have been easier!

4. Find the solution(s) to these equations

a. $x^2 - 10 = 3x$ b. $3 - 4x = 7x^2$ c. $2x^2 = (x+1)^2 + 1$

The axis of symmetry

As a result of the activities in this unit you may have reflected on the *symmetrical* nature of the graphs of quadratic functions. In other words you may have noticed that there is a vertical *line of symmetry* (sometimes thought of as a "mirror line") through the vertex/turning point of the graph of all quadratic functions. This vertical line is more formally known as the *axis of symmetry*.



If you recall, the vertex of $y = ax^2 + bx + c$ occurs at $\left(-\frac{b}{2a}, \frac{b^2-4ac}{4a}\right)$.

This means that the axis of symmetry must have the equation $x = -\frac{b}{2a}$.

The equation of the axis of symmetry can also be found very easily if the turning point is known. It can also be found if the location of two points with the same y value are known, e.g. the location of the zeros.

5. Find the equation of the axis of symmetry of the graph of

a. $y = 3x^2 + 12x + 1$ b. $y = 2(x-2)(x-8)$ c. $y = -(x-4)^2 + 50$

d. $y = x^2 + 3x + 6$ e. $y = \frac{1}{2}x^2 + 8x - 3$ f. $y = 5x^2 - 25$

6. For the following quadratic functions

i. Find the exact co-ordinates of the x and y axes intercepts
ii. Find the nature (max or min) and position of the turning point.
iii. Find the equation of the axis of symmetry
iv. Sketch its graph showing these features.

a. $y = x^2 + 3x - 10$ b. $y = -x^2 + 2x + 3$ c. $y = 2x^2 + 8x + 4$

14. Applying your Knowledge

$$y = ax^2 + bx + c$$

" … understand *a* … got *c* … but what's up with *b* ?.."

*By now, you should be able to describe the graphical significance of the co-efficients **a** and **c** in the function* $y = ax^2 + bx + c$.

*In other words, you should understand the effect that different values of **a** and **c** have on the graph of a quadratic function in general form. So, what about **b**?*

Your task

To investigate the graphical significance of ***b***, you are going to study the graphs of a number of quadratic functions where ***a*** and ***c*** are fixed and ***b*** takes a number of different values. For this, you are going to use the quadratic function

$$y = \quad x^2 + bx + \ldots$$

Insert your choice of c-value here!

Sketch

Select six or more values of ***b*** between -10 and 10. Include positive, negative, zero, integer and fractional values. Draw the graph of the function for each of these on the same A4-sized set of axes. Label each parabola clearly with its function.

Comment

Describe (in your own words, in sentences) the effect that altering the value of ***b*** has on the graphs of your family of quadratic functions.

In particular discuss what changes and what stays the same when ***b*** varies.

Explain, if you can, why some of this is so.

Tabulate

Draw up a table to summarise the key features of the graphs of the 6 + members of your quadratic family

Quadratic Function	*x-intercept*	*y-intercept*	*Co-ordinates of turning point*
…			

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Focus

Consider now the Turning Points of your family. Plot them on a new set of axes.

What do you notice?

Calculate the Turning Points of some other members of your family and add them to this graph.

Find an equation that describes the shape of this graph of the turning points.

Conclusion

Describe, in your own words, the effect of ***b***, on the graph of $y = ax^2 + bx + c$.

It may be useful to think about if ***b*** is zero and gets small (i.e. $b \to -\infty$) what happens to the graph and if ***b*** is zero and gets large (i.e. $b \to \infty$) what happens to the graph.

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15. The Discriminant – the tool of the maestro.

Recall that completing the square on the general quadratic function gives:

$$y = ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$$

and hence we can see that the vertex occurs at $\left(\frac{-b}{2a}, -\frac{b^2 - 4ac}{4a} \right)$

Let us now systematically consider the different numbers of zeros that a quadratic function may have.

We have seen that two zeros occur when the graph cuts the x-axis twice.
As described in Section 11, there are two possibilities here:

- the graph has a negative minimum value, i.e. $a > 0$ and $-\frac{b^2 - 4ac}{4a} < 0$
  which implies that the numerator is positive i.e. $b^2 - 4ac > 0$.
- the graph has a positive maximum value, i.e. $a < 0$ and $-\frac{b^2 - 4ac}{4a} > 0$
  which implies that the numerator is positive i.e. $b^2 - 4ac > 0$.

We have seen that exactly one zero occurs when the vertex of the graph lies on the x-axis. There are two possibilities here:

- the graph has a zero minimum value, i.e. $a > 0$ and $-\frac{b^2 - 4ac}{4a} = 0$
  which implies that the numerator is zero i.e. $b^2 - 4ac = 0$.
- the graph has a zero maximum value, i.e. $a < 0$ and $-\frac{b^2 - 4ac}{4a} = 0$
  which implies that the numerator is zero i.e. $b^2 - 4ac = 0$.

We have seen that no zeros occur when the graph does not cut the x-axis.
There are two possibilities here:

- the graph has a positive minimum value, i.e. $a > 0$ and $-\frac{b^2 - 4ac}{4a} > 0$
  which implies that the numerator is negative i.e. $b^2 - 4ac < 0$.
- the graph has a negative maximum value, i.e. $a < 0$ and $-\frac{b^2 - 4ac}{4a} < 0$
  which implies that the numerator is negative i.e. $b^2 - 4ac < 0$.

Hence we can see that the expression $b^2 - 4ac$ "discriminates" between quadratics with respect to the number of zeros that they have i.e.

* $b^2 - 4ac > 0$ implies that the quadratic has two zeros.
* $b^2 - 4ac = 0$ implies that the quadratic has one zero (sometimes described as "one repeated zero").
* $b^2 - 4ac < 0$ implies that the quadratic has no zeros.

For this reason $b^2 - 4ac$ is called the *discriminant* of the quadratic.

It is given the symbol Δ (the greek letter D) i.e. $\Delta = b^2 - 4ac$

It provides a quick and easy algebraic way of determining the number of zeros of a quadratic function.

1. Calculate the *discriminant* and hence determine the number of zeros of each of the following functions:

a.	$y = x^2 + 5x + 2$	b.	$y = 3x^2 + 6x + 3$	c.	$y = 4x^2 + 5x + 2$
d.	$y = -3x^2 + 2$	e.	$y = -x^2 + \frac{2}{3}x - \frac{2}{7}$	f.	$y = x^2 + mx + m^2$

2. For what value(s) of k does,

a. $y = x^2 + 2x + k$ have no zeros.

b. $y = kx^2 - x + 3$ have two distinct zeros.

c. $y = -x^2 - 3x + 2k$ have exactly one zero.

d. $y = x^2 + kx + 4$ have no zeros.

16. eTech Support.

16.1 Opening a geometry file and animating it.

Enter the `GEOM` mode of your *9860*.

Press `File` [F1] and then press `2:Open`.

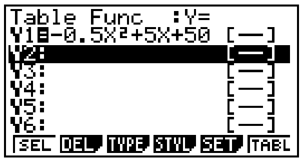
Move the input bar onto the file required and press [EXE].

Should you get it, do not worry about the message `Clear Current Image?`
– unless there are unsaved changes to a currently open Geometry file that you do not wish to lose.

With a file that is able to be animated (like the ones in this unit), the sequence required to run an animation is,

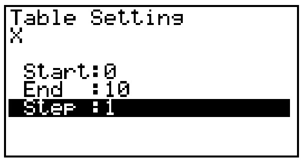
- go to the `Animate` menu by pressing [F6],
- chose either
  - `5:Go (once)` – for a single "run through" or
  - `6:Go (repeat)` – for repeated animation (stopped by [AC/ON])

16.2 Generating tabular representations of functions.



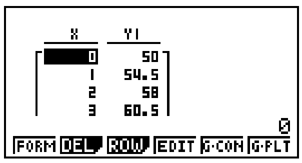
Enter the TABLE mode of your 9860.

Enter the function of interest into a free row. Make sure that you use [X,θ,T] to enter your variable x. Use [x^2] to square it.



To set the input values of x for which you would like the table drawn, press `SET` [F5].

Select the x value that will `Start` your table, `End` your table and the `Step` (or gap size) in between successive x values in your table. Press [EXE] after each entry.

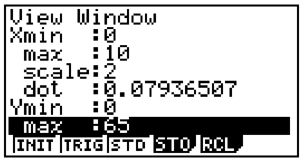


Press `TABL` [F6] to see a table of your chosen inputs and the corresponding outputs as determined by your function.

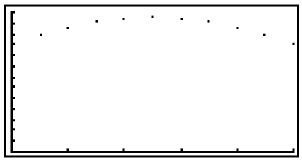
Use your Arrow Pad to move up and down the column of x values or across to the column of function values.

To plot the table's values on a Cartesian Plane press `G-PLT` [F6].
This will plot these values on the currently set View Window.
This will often need to be adjusted to incorporate the points under consideration.

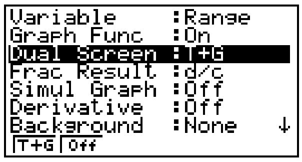
Adjusting the View Window.



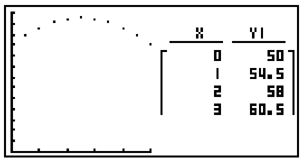
Press [SHIFT] then `V-WIN` [F3] to access the View Window settings. Enter as `Xmin` the least value on your x-axis and as `max` the greatest value on your x-axis.
Enter as the `scale` your choice of distance between the "ticks" on your x-axis.
Press [EXE] between settings. Set your y-axis in a similar way.
Press [EXE] again to finalise your settings. The plot of your table can now occur on the axes that you have just set up.



Viewing table and graph together.

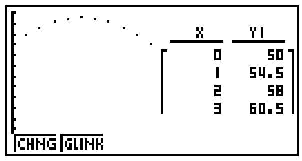


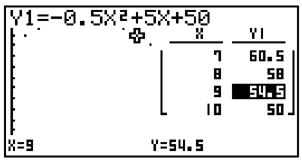
To see a table and its graph together,
enter the `SET UP` by pressing [SHIFT] and then [MENU],
arrow down ▼ to `Dual Screen` and choose `T+G` [F1]. Now when you G-PLT you will see your table and its graph side by side.



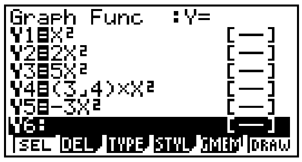
Linking table and graph.

When viewing a table and its graph in dual screen (as above), a link can be established between the pairs of values in your table and the corresponding points plotted in your graph. To establish this link press [OPTN] then `GLINK` [F2].
If you arrow up and down through the table a cursor will indicate corresponding points.





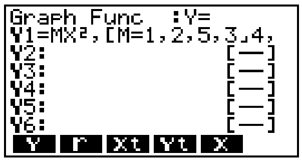
16.3 Generating graphical representations of functions.

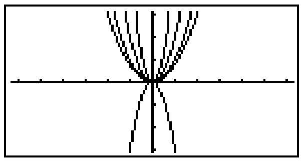


Enter the function(s) of interest into GRAPH mode of your 9860.
Upon entry, functions are automatically selected (indicated by the black box around their equals sign).
They can be deselected or reselected by pressing SEL [F1].

Prior to graph drawing it is wise to put some thought into the View Window settings (see 16.2). Once these are satisfactory press DRAW [F6].

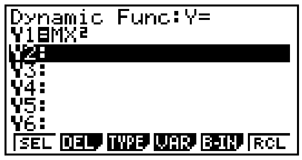
Viewing families of functions.





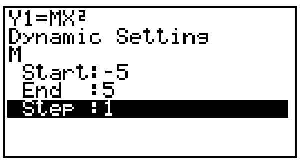
Families of functions can be entered using a single parameter, providing that values are assigned to the parameter as shown. The parameter (a letter of choice), square brackets, and equals sign are red [ALPHA] or yellow [SHIFT] entries above the keys on your 9860.
Use [,] for the commas. Either style of entry will result in multiple functions being drawn on the same axes.

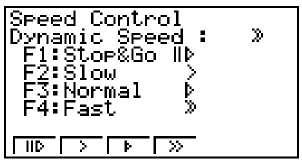
Viewing families of functions dynamically.



The effect of changing a parameter upon a function can be view dynamically in DYNA mode.
Enter a function in terms of a parameter and press VAR [F4].

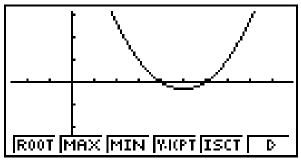
Press SET [F2] to select the values for your parameter, and [EXE] your settings.
Press SPEED [F3] to choose the speed of the animation. [EXE] your settings.
Press DYNA [F6] to see your family of functions represented dynamically.
Press [AC/ON] to stop an ongoing animation.





16.4 Obtaining graphical information about functions.

With a graph drawn in GRAPH mode, a wealth of graphical information can be obtained via the G-Solve menu which is obtained by pressing SHIFT and then G-SLV F5.



ROOT F1 will find the zero(s) of the function that are within the view window.

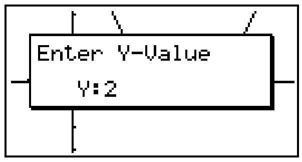
Max F2 will find any maximum turning points that are within the view window.

Min F3 will find any minimum turning points that are within the view window.

Y-ICPT F4 will find the y-intercept if it is within the view window.

ISCT F5 will find the intersection of two graphs that are drawn and meet within the view window.

Press F6 then Y-Cal F1 to calculate a y-value for your choice of x-value.

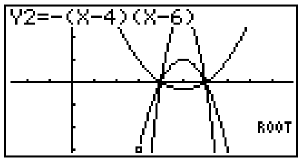


Press F6 then X-Cal F2 to calculate a x-value for your choice of y-value.

Notes on G-Solve.

Should more than one of the feature being sought occur in the view window (i.e. 2 zeros), the leftmost one will be found first. Press ▶ to move to the next one.

If more than one function is graphed, you will need to choose which graph for which features are being sought. Make this selection by pressing ▲ and ▼ and then EXE.



17 Answers

1.2. Investigating Areas

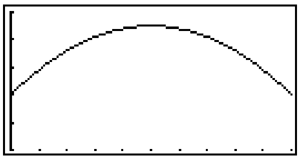
Qu.1

Find area of ΔPAQ and of ΔQBC and take these from the total area of $100 cm^2$.

Qu. 2

10	0	50	50
9	0.5	45	54.5
8	2	40	58
7	4.5	35	60.5
6	8	30	62
5	12.5	25	62.5
4	18	20	62
3	24.5	15	60.5
2	32	10	58
1	40.5	5	54.5
0	50	0	50

Qu.3



Qu.4

As AQ increases the area of PQCD increases, reaches a maximum, then decreases.

Qu.5

$62.5 cm^2$ when AQ=5 cm.

$50 cm^2$ when AQ=0 cm or 10 cm.

1.3. Capturing the ∞

Qu.1

Infinitely many

Qu.2

The length of AQ

Qu.3

$0 \le x \le 10$

Qu.4

$\frac{1}{2}x^2$

Qu.5

1	10 – 1 = 9
2	10 – 2 = 8
3	10 – 3 = 7
4	10 – 4 = 6
5	10 – 5 = 5
x	10 – x

Qu.6

$\frac{1}{2} \times (10 - x) \times 10$

$= 5(10 - x)$

Qu.7

$100 - \frac{1}{2}x^2 - 5(10 - x)$

$= 100 - \frac{1}{2}x^2 - 50 + 5x$

$= -\frac{1}{2}x^2 + 5x + 50$

Qu.8

$x = 0$ and $x = 10$

Qu.9

This equation is finding when the area of PQCD is equal to 50 cm and its solutions are the corresponding lengths of AQ.

4. The road to proof

Qu.4

	max /min	of y=	when x =
a	min	– 6	– 2
b	max	14	4
c	min	≈ -1.2	≈ -6.1
d	min	5	– 3
e	max	– 7	2
f	max	2	– 8

Qu. 6

	max /min	of y=	when x =
a	min	11	– 1
b	min	– 15	2
c	min	– 9	– 12
d	max	– 1	4
e	max	– 9	3
f	min	0	3
g	min	1	7
h	min	– 11	– 3

The proof requires an argument like (for a):

$(x+1)^2 \ge 0$

$\therefore (x+1)^2 + 11 \ge 11$

$\therefore y \ge 11$

This min. value of y = 11 occurs when

$(x+1)^2 = 0$

$\therefore x = -1$

5. Forms of Quad. Fns

Qu.1

$y = ax^2 + 2ahx + ah^2 + k$

a, $2ah$ and $ah^2 + k$ are three constants in

$y = \ldots x^2 + \ldots x + \ldots$

Qu.2

a. $y = x^2 + 6x + 11$

b. $y = x^2 - 4x + 5$

c. $y = x^2 - 10x + 5$

d. $y = -x^2 - 2x - 5$

e. $y = 2x^2 - 24x + 22$

f. $y = -3x^2 - 24x - 36$

6. Turning Point form

Qu.1

a. $y=(x+4)^2-17$
min $y=-17$
when $x=-4$

b. $y=(x+5)^2-23$
min $y=-23$
when $x=-5$

c. $y=(x-1)^2+1$
min $y=1$
when $x=1$

d. $y=(x-2)^2-10$
min $y=-10$
when $x=2$

e. $y=(x+\frac{3}{2})^2-\frac{9}{4}$
min $y=-\frac{9}{4}$
when $x=-\frac{3}{2}$

f. $y=(x-\frac{5}{2})^2+\frac{135}{4}$
min $y=\frac{135}{4}$
when $x=\frac{5}{2}$

7. Further C.T.S.

Qu.1

a. $y=2(x+2)^2-12$

b. $y=5(x+1)^2-2$

c. $y=-2(x-\frac{3}{2})^2+\frac{13}{2}$

d. $y=-3(x+2)^2+13$

e. $y=-2(x-\frac{5}{4})^2-\frac{31}{8}$

f. $y=\frac{1}{2}(x+3)^2+\frac{1}{2}$

g. $y=\frac{1}{3}(x-18)^2-102$

h. $y=-\frac{1}{2}(x-2)^2+4$

i. $y=a(x+\frac{b}{2a})^2-\frac{b^2-4ac}{4a}$

8. The graphs of quadratic. functions

8.1 Symmetry

Qu.1

a. Gets big, fast.
(grows faster as x decreases)

b. Gets big, fast.
(grows faster as x increases)

c. The graph will get very 'steep'.
As the behaviour is the same, the graph will be *symmetrical* either side of $x=0$

Qu.2

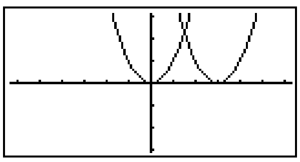
a. It gets smaller as x approaches 3, then it gets big, fast as x exceeds 3.

b. Gets big, fast.

c. The graph will have its lowest point when x =3. It will have similar 'steepness' to $y=x^2$ either side of $x=3$.

Qu.3

x	$y=x^2$	$y=(x-3)^2$
– 2	4	25
– 1	1	16
0	0	9
1	1	4
2	2	1
3	9	0
4	16	1
5	25	4

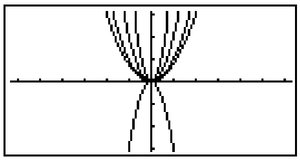


Qu.4

The same each side of a central/mirror line.

8.2 Families of Q.F.

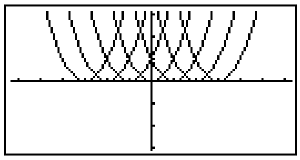
Qu.1



(i) Same parabolic shape.
A shared max/min point of (0,0).
All symmetrical either side of $x=0$.

(ii) Some are 'tighter' that others i.e. some grow faster than others.

(iii) $y=-3x^2$ is an 'odd one out' as it is 'up-side down' compared to the others.



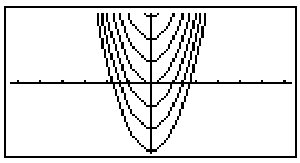
Qu 2 (a) $y=(x-h)^2$

(i) Exactly the same shape.
Min value at (0,h)

(ii) Position of min.
Moved across h units left (h>0) or right (h<0)

(iii) No.

Qu 2 (b) $y=x^2+k$

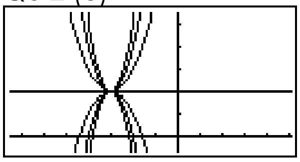


(i) Exactly the same shape.
Min value at (k,0)

(ii) Position of min.
Moved up/down k units up (k>0) or down (k<0)

(iii) No.

Qu 2 (c)



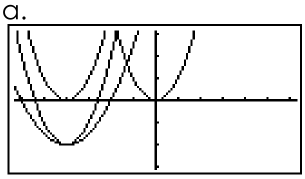
(i) Same parabolic shape.
A shared max/min point of (-3,2).
All symmetrical either side of $x=-3$.

(ii) Some are 'tighter' that others i.e. some 'grow' faster than others.

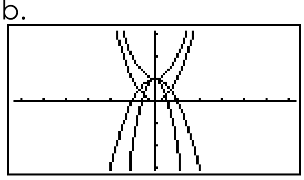
(iii) $y=0(x+3)^2+2$ is an 'odd one out' as it is a straight line and not a parabolic shape. This is because it is really just $y=2$.

Qu 3

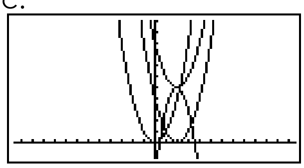
a.



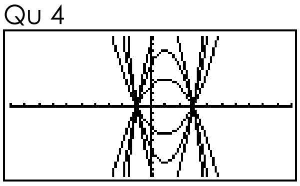
b.



c.



Qu 4



They all share the x-intercepts of -1 and 3. This is because

$$y=mx^2-2mx-3m$$

$$y=m(x^2-2x-3)$$

and the values $x=-1$ and $x=3$ make $x^2-2x-3=0$ making the function zero regardless of the value of the parameter.

8.3 Find a function…

(some possible answers)

Qu.1

$$y=x^2+5$$

$$y=-x^2-100$$

$$y=(x+4)^2+1$$

Qu 2

$$y=x^2$$

$$y=(x-20)^2$$

Qu 3

$$y=(x-3)(x+4)$$

$$y=-2(x+1)(x+12)$$

Qu 4

I don't think so..

$$y=(x+3)(x-2)(x+5)$$

will work, but is it a quadratic function?

Qu. 5

a.

$$y=(x-5)^2+13$$

$$y=-4(x-5)^2+13$$

$$y=-\tfrac{13}{16}(x-1)(x-9)$$

b.

$$y=-4(x-5)^2+13$$

$$y=-\tfrac{13}{16}(x-1)(x-9)$$

c.

$$y=\tfrac{13}{4}(x-3)^2$$

$$y=\tfrac{13}{36}(x+1)^2$$

d.

$$y=-(x-6)^2+14$$

$$y=-(x-4)^2+14$$

9. So, how am I going?

Qu. 1

a. $y=x^2-14x+25$

b. $y=-2x^2+4x+4$

Qu. 2

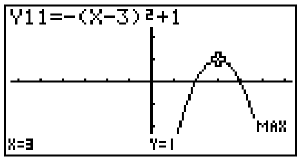
a. $y=(x+5)^2+5$

b. $y=4(x+\tfrac{3}{2})^2-4$

Qu. 3

a. $(3,1)$

b.



Qu 4

	$y=(x+2)^2+11$	$y=-5(x-3.5)^2+2$
max/ min	min	max
Of	11	2
When	x = 2	x = 3.5
no. of zeros?	0	2

Qu 5

a. The graph is tightened / narrowed.

b. The graph is moved up 5 units.

Qu. 6

a. $y=-\tfrac{1}{3}(x-3)^2+13$

b.

$(x-5)^2 \geq 0$

$\therefore -\frac{1}{2}(x-5)^2 \leq 0$

$\therefore -\frac{1}{2}(x-5)^2 + 62.5 \leq 62.5$

Qu. 7

a. (i) 6 cm

(ii) 20 cm^2

(iii) 30 cm^2

(iv) 50 cm^2

b. (i) 10 – x cm

(ii) 5x cm^2

(iii) 5(10 – x) cm^2

(iv) 5x+5(10 – x) cm^2

c. it is constant (with a value of 50 cm^2) as

$5x+5(10-x)$

$=5x+50-5x$

$=50$

10. The Turning Point

Qu.1

a. $(-\frac{3}{2},-\frac{25}{4})$

b. $(-\frac{2}{5},-\frac{14}{5})$

c. $(\frac{3}{2},\frac{17}{4})$

d. $(-2,1)$

e. $(\frac{1}{6},-\frac{29}{126})$

f. $(-1,k-2)$

12. What sort of zeros?

Qu.1

No zeros

No zeros

$x=2$

$x=1$ and $x=3$

$x=0.586$ & $x=3.414$

$x=0.268$ & $x=3.732$

$x=0$ & $x=4$

$x=-0.236$, $x=4.236$

$x=-0.449$, $x=4.449$

$x=-0.646$, $x=4.646$

$x=-0.828$, $x=4.828$

$x=-1$ and $x=5$

Qu. 2

a.

- They vary in number.
- Some are integer, some are not.
- They are equal in distance to $x=2$.

b.

An infinite number, as there are infinite pairs of integers equidistant to $x=2$.

c.

$y=x^2-4x-12$

Zeros: $x=-2$, $x=6$

d.

$y=x^2-4x-21$

Zeros: $x=-3$, $x=7$

$y=x^2-4x-32$

Zeros: $x=-4$, $x=8$

$y=x^2-4x-45$

Zeros: $x=-5$, $x=9$

$y=x^2-4x-60$

Zeros: $x=-6$, $x=10$

e.

As the graph moves down it will not always pass through integer points on the x-axis.

Qu.3

a.

$(-2)^2-7(-2)-18$

$=4+14-18$

$=0$

$(-2+2)(-2+9)$

$=0\times-11$

$=0$

$x=9$

$(9)^2-7(9)-18$

$=81-63-18$

$=0$

$(9+2)(9-9)$

$=11\times0$

$=0$

Qu.4

a. $y=(x-5)(x+2)$

So the zeros are $x=5$ and $x=-2$

b. $y=(x-1)(x-1)$

So the zeros are $x=1$ (and $x=1$?)

c. $y=(x-1)(x+6)$

So the zeros are $x=1$ and $x=-6$

d. $y=(x+6)(x-5)$

So the zeros are $x=-6$ and $x=5$

e. $y=(2x+3)(x+3)$

So the zeros are $x=-\frac{3}{2}$ and $x=-3$

f. $y=(x-1)(3x+14)$

So the zeros are $x=1$ and $x=-\frac{14}{3}$

g. $y=-(x+10)(x+1)$

So the zeros are $x=-10$ and $x=-1$

h. $y=-(x+5)(4x-1)$

So the zeros are $x=-5$ and $x=\frac{1}{4}$

Qu.5

a. $y=(x+1)(x-2)$

$= x^2 - x - 2$

b. $y = (2x+2)(x-2)$

$= 2x^2 - 2x - 4$

c. An infinite number

d. $y = a(x+1)(x-2)$

$= ax^2 - ax - 2a$

providing that $a \neq 0$

Qu.6

a. $y = (x-4)(x-6)$

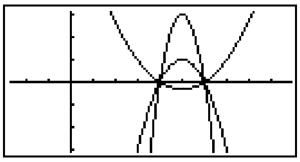
b. $y = a(x-4)(x-6)$

providing that $a \neq 0$

c. $y = \frac{1}{3}(x-4)(x-6)$

d. $y = -(x-4)(x-6)$

e. $y = -3(x-4)(x-6)$



13. Solving Quadratics

Qu.1

a. $(5,-4)$

b. $(3,0)$

c. $(1,-3)$

d. $(\frac{3}{2},\frac{13}{4})$

e. $(-2,-32)$

f. $(-\frac{1}{3},0)$

g. $(\frac{1}{2},-20\frac{1}{4})$

h. $(2,-4)$

i. $(1,9)$

Qu.2

a. $x = -7$ and $x = -3$

b. $x = 3$

c. $x = 2+\sqrt{5}$ and

$x = 2-\sqrt{5}$

(approximately)

$x = -0.732$ and

$x = 2.732$

d. $x = 2$ and $x = -5$

e. $x = -6$ and $x = 2$

f. $x = -\frac{1}{3}$

g. $x = 5$ and $x = -4$

h. No zeros

i. $x = -\frac{1}{2}$ and $x = \frac{5}{2}$

Qu.3

a. Turning point form

b. Factored form

c. General form

Qu.4

a. $x = -2$ and $x = 5$

b. $x = -1$ and $x = \frac{3}{7}$

c. $x = 1-\sqrt{3}$ and

$x = 1+\sqrt{3}$

Qu.5

a. $x = -2$

b. $x = 5$

c. $x = 4$

d. $x = -\frac{3}{2}$

e. $x = -8$

f. $x = 0$

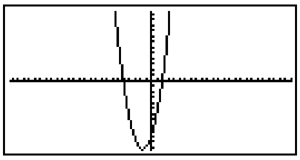
Qu.6

a. (i) $(-5,0)$ and $(2,0)$

$(0,-10)$

(ii) min at $(-\frac{3}{2},-12\frac{1}{4})$

(iii) $x = -\frac{3}{2}$

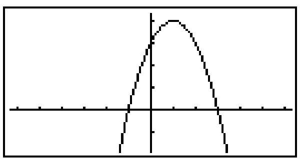


b. (i) $(-1,0)$ and $(3,0)$

$(0,3)$

(ii) max. at $(1,4)$

(iii) $x = 1$



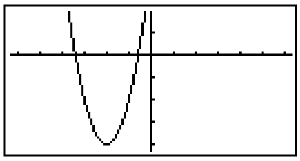
c. (i) $(-2+\sqrt{2},0)$ and

$(-2-\sqrt{2},0)$

$(0,4)$

(ii) min. at $(-2,-4)$

(iii) $x = 2$



15. The Discriminent

Qu.1

a. $\Delta = 17$
so there are 2 zeros.

b. $\Delta = 0$
so there is 1 zero.

c. $\Delta = -7$
so there are no zeros.

d. $\Delta = 24$
so there are 2 zeros.

e. $\Delta = \frac{44}{63}$
so there are 2 zeros.

f. $\Delta = -3m^2$
so there are no zeros.

Qu.2

a. $k > 1$

b. $k < \frac{1}{12}$

c. $k = -\frac{9}{8}$

d. $-4 < k < 4$

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