Mental-jaunt #1

(Feb 14, 2013 ❤)

Halving, in mathematics, means to divide by 2. Halving is not "harfing".

Let's define "Harfing" as taking an integer with an even number of digits and separating them (centrally) into two digits (which are called the 'harfs')

e.g. the result of harfing 73 is 7 and 3 and the result of harfing 4580 is 45 and 80.

a) Can you find an integer for which squaring the sum of its harfs gives the integer?

b) If you find one can you find more?

c) If you do, what questions are you now wondering about?

d) Most importantly, share the mental-jaunt you went on with someone - explaining what you were thinking at the very start and how you reached your destination.

One person's account of their journey

I started by checking what happens with 2-digit numbers.

e.g. 27 → 2 and 7 → 9 → 81

I tried a few of these and ended up trying to find what numbers got close to working.

Then, ***it occurred to me*** that the number I started with had to be a square number! So all I had to check in the 2-digit range was 16, 25, 36, 49, 64 and 81.

Checking them revealed 81 → 81 and I was 100% sure I had found all two digit "Harfys".

This could well be a suitable stopping point for some students.

Then I wondered about 4-digit numbers and immediately wondered how many 4 digit square numbers there were.
With the help of my mind I knew that 30^3 was 900 and
then with the help of my trusty calculator I reckoned that 32^2 gave the smallest 4 digit square (1024) and
with my mind I reckoned that 100^2 was the first 5 digit square
checking with my calculator revealed that 99^2 (9801) was the largest 4-digit square

So I had the squares between 1024 and 9801 to check, but how many are there?
99 – 31 = 68

A spreadsheet hunt revealed (downloadable from where you got this document) 2025, 3025 and 9801 and so I am 100% certain there are three 4-digit *Harfys*.

This could well be a suitable stopping point for some students.

My next thought was 'is there an algebraic way to attack this?" And yes, there is, but I will not lay it out here. You can find one at
http://answers.yahoo.com/question/index?qid=20071113225137AA2pt3B

My next thought was well what about 6-digit numbers?

Which quickly lead me to the realization that then there was 8, then there was 10, ……. Which lead me think whether or not there was a completely general way to generate such numbers, independent of the number of digits?

Ooooh! *This could well be a suitable stopping point for some students (like me).*

That reminded me of the 400 problem once shown to me by Derek Holton, which gave me hope there may be a solution: http://nzmaths.co.nz/resource/400-problem?parent_node=

That is where my journey ended – at least for the time being.

It is OK to stop!

If you go back through this account and think about teach step, I hope you would agree with me that this type of thinking is valuable and key to being able to understand more challenging concepts in mathematics. If our students are helped to develop this way of thinking, I think it can only be a positive thing.

Students need time to ponder-at this sort of task. It is not something for the last 10 minutes of a lesson, they need to ponder over time and share the journey they took and as a teacher, certainly if the students are inexperienced as this, you need to lead them along the way and then recap one way it can be approached.

Most importantly, being involved in this sort of thing is, IMHO, how ***things occur to us*** at a later date, when trying to makes sense of other things.

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